Adenosine Triphosphate as a Function of pH, Mg^{2+} Ion Concentration, and Ionic Strength," *J. Biol. Chem.*, **244**, 3330 (1969).
Robinson, S. M., "Interpolative Solution of Systems of Nonlinear Equations," *SIAM J. Numer. Anal.*, 3, 650 (1966).
Rose, I. A., "The State of Magnesium in Cells as Estimated from the Adenylate Kinase Equilibrium," *Proc. Natl. Acad. Sci.*, **61**, 1079 (1968).
———, M. Grunberg-Manago, S. R. Korey, and S. Ochoa, "Enzymatic Phosphorylation of Acetate," *J. Biol. Chem.*, **211**, 734 (1954).
Satchell, D. P. N., and G. F. White, "Kinetic Studies with Acetate Kinase," *Biochim. Biophys. Acta*, **212**, 248 (1970).
Sigel, H., K. Becker, and D. B. McCormick, "Ternary Complexes in Solution. Influence of 2,2′ Bipyridyl on the Stability of 1:1 Complexes of Co^{2+}, Ni^{2+}, Cu^{2+} and Zn^{2+} with Hydrogen Phosphate, Adenosine 5′-Monophosphate and Adenosine 5′-Triphosphate," *ibid.*, **148**, 655 (1967).
Smith, R. M., and R. A. Alberty, "The Apparent Stability Constants of Ionic Complexes of Various Adenosine Phosphates with Monovalent Cations," *J. Phys. Chem.*, **60**, 180 (1956).
Stock, D. I., and C. W. Davies, "The Second Dissociation Constant of Magnesium Hydroxide," *Trans. Faraday Soc.*, **44**, 856 (1948).
Taqui Khan, M. M., and A. E. Martell, "Metal Chelates of Adenosine Triphosphate," *J. Phys. Chem.*, **66**, 10 (1962).
———, "Thermodynamic Quantities Associated with the Interaction of Adenosinediphosphoric and Adenosinemonophosphoric Acids with Metal Ions," *J. Am. Chem. Soc.*, **89**, 5585 (1967).
Weitzel, G., and T. Spehr, "Zinc Complexes of Purines, Nucleosides and Nucleotides," *Z. Physiol. Chem.*, **313**, 212 (1958).
Williamson, J. R., and E. B. Corkey, "Assays of Intermediates of the Citric Acid Cycle and Related Compounds by Fluorometric Enzyme Methods," in *Methods in Enzymology*, S. P. Colowick and N. O. Kaplan, ed., Vol. 3, p. 434, Academic Press, New York (1969).

Manuscript received July 28, and accepted October 8, 1976.

A Comprehensive Correlating Equation for Laminar, Assisting, Forced and Free Convection

A correlating equation for assisting convection was developed by combining correlating equations for pure free and pure forced convection. These component equations are based on laminar boundary-layer theory for an isothermal, vertical plate. Theoretical values for assisting convection indicate that the third root of the sum of the third powers gives the best representation, as contrasted with the choice and rationalization of the second or fourth power by prior investigators.

This expression was modified by the addition of a limiting value Nu_o to obtain a better representation below the domain of boundary-layer theory and was generalized for uniform heating and for spheres and horizontal cylinders by the appropriate choice of the characteristic length.

STUART W. CHURCHILL

Department of Chemical and Biochemical Engineering
University of Pennsylvania
Philadelphia, Pennsylvania 19174

SCOPE

Heat transfer by forced convection implies a temperature difference and hence a density difference. The density difference gives rise to free convection. This coupled process has been the subject of both experimental and theoretical investigations for a variety of geometries and conditions. A number of different correlating equations have been proposed for particular cases. However, the accuracy, generality, and range of validity of these correlations have not been tested critically.

The objective of this work has been to develop a more general and accurate correlating equation for combined convection. Attention is confined to assisting convection, that is, to forced flow in the same direction as the buoyant motion, and to the laminar regime but includes the effects of the Prandtl number, boundary conditions, and shape.

The correlating equation is based on theoretical results for free convection, forced convection, and combined convection for vertical plates but was tested with experimental data for plates, spheres, and horizontal cylinders.

CONCLUSIONS AND SIGNIFICANCE

Equation (18), together with supplementary Equations (11) and (12) and the coefficients in Table 1, provide a good representation for the experimental data for both the local and overall Nusselt number for combined forced and free convection from a vertical plate over the entire potential flow and laminar boundary-layer regimes ($Re < 10^4$, $Ra < 10^9$) for all Prandtl numbers and for both uniform surface temperature and uniform heating. This expression appears to be valid for spheres and horizontal cylinders if the indicated characteristic lengths are used. It appears to be applicable for other shapes, such as wedges and vertical cylinders, based on the fragmentary values which are available.

Powers of 2 and 4 rather than the third power of Equation (18) were chosen for correlation by earlier workers, apparently because of physical rationalizations, imprecise data, uncritical tests, and insensitivity of this type of expression to the value of the exponent.

Equation (18) reduces to Equation (10) for the laminar boundary-layer regime ($15 < Re < 10^4$, $2\,300 < Ra < 10^9$). Equation (10) is based wholly on theoretical values.

For very small wires, such that $RePr$ and $(RaPr)^3$ are both less than $\pi/5$, Equation (17), based on potential flow, may be more reliable than Equation (18). Also, the experimental results of Gebhart and Pera (1970) for small wires reveal a significant dependence on L/d in this regime, presumably due to three-dimensional effects.

Equations (18), (10), and (17) are presumed to be valid for component transfer with Sh, Sc, and Ra' substituted for Nu, Pr, and Ra, although confirmatory data are not available.

Additional theoretical solutions for spheres and horizontal cylinders are needed to provide firm expressions for the dependence of free and forced convection on Pr and to test Equations (10) and (18) for combined convection.

Some experimental studies of forced, free, and combined convection from spheres have yielded values of Nu and Sh far greater than indicated by Equation (18). This discrepancy is worthy of resolution and explanation.

Presentation of tabulated values of the individual dimensionless groups, Nu, Re, Pr and Gr or Ra, is desirable in both theoretical and experimental studies. Graphical correlation of combined groups makes their accurate inclusion in subsequent correlations difficult or impossible.

Attempts to generalize the correlating equation in some simple fashion to include forced convection oblique to or opposed to free convection were unsuccessful and, as suggested by Weber (1976), may not be feasible.

PREVIOUS WORK

McAdams (1942) simply recommended the use of the higher of the heat transfer coefficients for free and forced convection alone for the combined process. However, most of the correlating equations that have subsequently been proposed have utilized the sum of some arbitrary power of correlating equations for free and forced convection, namely

$$Nu^n = Nu_F{}^n + Nu_N{}^n \qquad (1)$$

where the subscripts F and N indicate forced and free (natural) convection, respectively. [Equation (1) has the same form as the general expression proposed by Churchill and Usagi (1972) for correlation, if Nu_F and Nu_N are interpreted as the asymptotic solutions for Ra/Re^2 approaching zero and infinity.]

Many other correlating equations have been based on the specification of an equivalent forced velocity for free convection. Lemlich and Hoke (1956) demonstrated that this concept can be used to superimpose the correlations for pure free and pure forced convection to cylinders. However, the choice of an equivalent velocity is somewhat arbitrary. In addition, an arbitrary combining rule for the imposed and equivalent velocities is necessary for mixed convection. The usual combining rule for this case is again the n^{th} root of the n^{th} power of the components, namely

$$Nu^n = (E\,Re^p)^n + (F\,Gr^q)^n \qquad (2)$$

where p and q for the laminar boundary-layer regime are ½ and ¼, respectively, and E and F are functions of Pr. Equation (2) can be recognized as a special case of (1). The various prior correlating equations for assisting convection can be reviewed concisely in terms of these two models.

Krischer and Loos (1958) postulated that Re in graphical correlations for forced convection from air to various bodies be replaced by $Re + (Gr/2)^{1/2}$ which is equivalent to using $pn = 1$, $qn = 1/2$ and $E/F = 2^{1/2n}$ in Equation (2). If $p = 1/2$ and $q = 1/4$, $n = 2$ and $E/F = 2^{1/4}$. Brdlik et al. (1974) postulated velocity and temperature distributions to derive an expression for Nu_x for vertical plates. This expression was then utilized by an unexplained procedure to obtain

$$(\overline{Nu}/0.68)^2 = (0.952 + Pr)^{1/2} Gr^{1/2} \pm PrRe \qquad (3)$$

with the plus sign for assisting and the minus sign for opposing flow. Equation (3) corresponds to Equation (2), again with $n = 2$, in terms of Re and Gr but yields the wrong dependence on Pr for large Re and Pr and for large Gr and small Pr.

Other investigators have generally chosen $n = 4$. Börner (1965) correlated data for flat plates, cylinders, and spheres, with assisting and opposing convection, by adding Re and $(Gr/2)^{1/2}$ vectorially. For assisting convection this yields $n = 4$, but again $E/F = 2^{1/4}$. Acrivos (1966) used Equation (1) for correlation of his computed values for assisting convection near the forward stagnation point of horizontal cylinders. He chose an exponent of 4 to yield a satisfactory numerical approximation of the more complex theoretical dependence. Hatton et al. (1970) added vectorially Re_d and an effective Re_d for free convection from electrically heated cylinders with the effective Re_d obtained by equating the correlations for pure forced and pure free convection. The result for assisting flow is equivalent to Equation (2) with $n = 4$. Jackson and Yen (1971) recorrelated the data of Oosthuizen and Maden (1970) for assisting flow of air over heated horizontal cylinders using the equivalent of Equation (2) with $p = 1/2$, $q = 1/4$, $n = 4$, and $E/F = 1$. They rationalized the exponent of 4 on the basis of the additivity of the work of free and forced convection. Oosthuizen and Bassey (1973) used the same exponent for their data for assisting flow of air over heated vertical plates but added a constant term to both of the boundary-layer expressions to account for the behavior in the conductive regime.

A slight modification of Equations (1) and (2) leads to the correlation of the effect of free convection superimposed on forced convection, or vice versa, in terms of the group Gr/Re^2, in an asymmetric form. Thus Sparrow and Gregg (1959) proposed the expression

$$\frac{Nu_x}{Nu_{xF}} = 1 \pm Q\{Pr\}\,\frac{Gr_x}{Re_x{}^2} \qquad (4)$$

to represent the computed behavior for vertical plates at $Gr_x/Re_x{}^2 << 1$, with the plus sign for assisting and the minus sign for opposing flow. Eshghy (1964) proposed the corresponding expression

$$\frac{Nu}{Gr^{1/4}} = Q_1\{Pr\}\left[\,1 + Q_2\{Pr\}\,\frac{Re}{Gr^{1/2}}\right] \qquad (5)$$

for $Re/Gr^{1/2} << 1$. Yuge (1960) correlated the deviations of his data for mixed convection with air and spheres from the limiting correlations as exponential functions of $0.493Re_d^{1/2} - 0.392Gr_d^{1/4}$. [However, Klyachko (1963) contends that these data are in error on the high side owing to movement of air in the laboratory. He then proposes a very complicated set of equations for the correlation of other data.] Oosthuizen and Madan (1970) used a three-term power series in Gr_d/Re_d^2 to correlate their data for the assisting flow of air over heated cylinders. Fand and Keswani (1973) derived separate dependences on Gr_d/Re_d^2 depending on the range of Gr_d/Re_d^2 for their data for water and a heated cylinder. For $0.5 < Gr_d/Re_d^2 \leq 4$, they added the term $0.033(Gr_d/Re_d^2)^{0.3}Gr_d^{1/4}$ to the terms for forced convection. For $2 < Gr_d/Re_d^2 < 40$, they used an expression equivalent to Equation (2) with $n = 2$, $p = 1/2$, and $q = 1/4$. Nakai and Okazaki (1975) proposed correlations for the deviations from the limiting correlations for very small cylinders in terms of the group $PrRe_d^3/\overline{Nu}_dGr_d$. Gebhart and Pera (1970) noted for small wires that the effect of superimposed free convection depended on $B\ Gr/Re_d^{p/q}$, with p/q and B varying with Pr and L/d. The ratio p/q varied from 2 to 4.

Morgan (1975) recently reviewed convection to cylinders. However,his attention to assisting convection is essentially limited to the conditions that produce a 5% increase in $\overline{Nu}$ above the value for forced convection alone.

In summary, Equation (1) appears to be generally satisfactory for correlation for combined convection. However, some uncertainty exists concerning the component parts, particularly with respect to Pr, the best value of the exponent n, and the behavior for very small cylinders.

Solutions and correlating equations for pure forced and pure free convection, and experimental and theoretical values for assisting convection, will be discussed below in connection with the development of the new correlation.

GENERAL DEVELOPMENT

The development herein consists of selection of correlating equations for pure free and pure forced convection to isothermal, vertical plates; a test of the suitability of Equation (1) for combined convection; evaluation of the exponent n in Equation (1) using theoretical values for assisting convection; generalization of the correlation for other boundary conditions and shapes; and comparison of the resulting expression with experimental values.

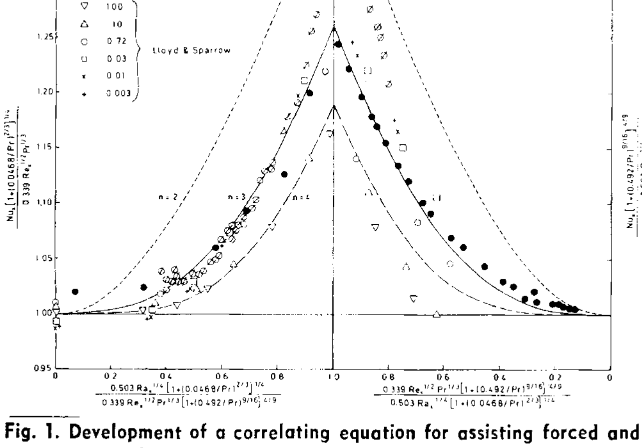

Fig. 1. Development of a correlating equation for assisting forced and free convection from an isothermal plate.

The initial development is carried out for isothermal vertical plates because of the availability of analytical solutions for pure convection for the limiting cases of large and small Pr and of reliable theoretical values for intermediate Pr.

LAMINAR BOUNDARY-LAYER REGIME ON VERTICAL PLATES

Uniform Wall Temperature

Churchill and Ozoe (1973*b*) and Churchill and Usagi (1972) developed and confirmed the accuracy of the following correlating equations for the local rate of heat transfer by pure free and pure forced convection in the laminar boundary-layer regime:

$$Nu_{xF} = 0.339\, Re_x^{1/2}Pr^{1/3}/[1 + 2(0.0468/Pr)^{2/3}]^{1/4} \quad (6)$$

$$Nu_{xN} = 0.503\, Ra_x^{1/4}/[1 + (0.492/Pr)^{9/16}]^{4/9} \quad (7)$$

These expressions are therefore postulated as the component parts of Equation (1). In order to test Equation (1), it is convenient to rearrange it in the following two forms.

$$\left(\frac{Nu}{Nu_F}\right)^n = 1 + \left(\frac{Nu_N}{Nu_F}\right)^n \quad (8)$$

and

$$\left(\frac{Nu}{Nu_N}\right)^n = 1 + \left(\frac{Nu_F}{Nu_N}\right)^n \quad (9)$$

Figure 1 is a plot of the several sets of theoretical values for combined convection in terms of the groupings in Equations (8) and (9) and the details provided by Equations (6) and (7). The symmetry of Equations (8) and (9) suggests the use of these split coordinates. The split coordinates permit the use of arithmetic scales and hence the display of absolute deviations. Indeed, this plot provides such a critical test that the computed, theoretical values, as tabulated in the original references, demonstrate some scatter. Nu_x/Nu_{xF} is plotted vs. Nu_{xN}/Nu_{xF} on the left side of Figure 1 and Nu_x/Nu_{xN} vs. Nu_{xF}/Nu_{xN} on the right side. Curves corresponding to $n = 2$, 3, and 4 are included. The theoretical values follow these curves, confirming the applicability of Equation (1) for assisting convection. The precise values of Merkin (1969) for $Pr = 1$ are represented by the curve for $n = 3$ within 1% except for two odd points. The values of Szewczyk (1964) for $Pr = 0.72$ agree equally well with the curve for $n = 3$ for small Ra_x but are 5% high at higher Ra_x. Merkin also noted this discrepancy in Szewczyk's values. The values of Lloyd and Sparrow (1970) for $Pr \leq 0.72$ generally follow this same curve, but the values for $Pr = 10$ and 100 are more closely represented by the curve for $n = 4$. On balance, a value of 3 appears to be the best choice for n. The resulting correlation can be written as

$$\left(\frac{Nu_x f_F\{Pr\}}{A_F Re_x^{1/2}Pr^{1/3}}\right)^3 = 1 + \left(\frac{A_x Ra_x^{1/4} f_F\{Pr\}}{A_F Re_x^{1/2}Pr^{1/3}F_N\{Pr\}}\right)^3 \quad (10)$$

with

$$f_F\{Pr\} = [1 + (C_F/Pr)^{2/3}]^{1/4} \quad (11)$$

$$f_N\{Pr\} = [1 + (C_N/Pr)^{9/16}]^{4/9} \quad (12)$$

Equations (10), (11), and (12) have a sufficiently general form to apply to the mean as well as the local Nusselt number of a vertical plate, to other boundary conditions, and to other shapes. For example, an expression for $\overline{Nu}$ in the form of Equation (10) can be obtained simply by integrating h in Equations (6) and (7) with respect to x

TABLE 1. RECOMMENDED CONSTANTS AND CHARACTERISTIC LENGTHS

			Values from boundary-layer theory			
	l	A_F	A_N	C_F	C_N	Nu_o
Vertical plate						
local						
uniform T	x	0.339	0.503	0.0468	0.492	0
uniform j	x	0.464	0.563	0.0205	0.437	0
mean						
uniform T	H	0.677	0.670	0.0468	0.492	0
uniform j	$H/2$	0.656	0.669	0.0205	0.437	0
Horizontal cylinder						
mean						
uniform T	πd	1.08[1]	0.690	0.412	0.559	1.0[2]
uniform j	πd	—	0.694	0.442	—	—
Sphere						
mean						
uniform T	$\pi d/2$	0.69[1]	0.659	—	—	π[3]
General approximate values	—	0.67[4]	0.67[5]	0.45[6]	0.45	—[7]

Notes
1—experimental
2—from the solution of King (1914) for potential flow
3—from the solution for pure conduction
4—except for Nu_x and cylinders
5—except for Nu_x
6—except use 0.035 for vertical plates
7—use 0.5 for vertical plates, 1.0 for cylinders and π for spheres

from 0 to H and repeating the rest of the derivation. The corresponding values of A_F and A_N are given in Table 1; n, C_F and C_N do not change. Derivations for another boundary condition and for other shapes follow.

Uniform Heat Flux

Churchill and Ozoe (1973*a*, 1973*c*) developed the following correlating equations for uniform heat flux analogous to Equations (6) and (7) for uniform temperature:

$$Nu_{xF} = 0.464\, Re_x^{1/2} Pr^{1/3} / [1 + (0.0205/Pr)^{2/3}]^{1/4} \quad (13)$$

$$Nu_{xN} = 0.563\, Ra_x^{1/4} / [1 + (0.437/Pr)^{9/16}]^{4/9} \quad (14)$$

Figure 2 is a plot of the computed values for uniform heat flux in the same form as Figure 1. The precise values of Wilks (1974) for $Pr = 1.0$ are generally represented within 1% by the curve for $n = 3$. Wilks (1973) earlier values for different Pr are also represented by this curve, although a few values scatter, probably due to numerical error. Hence, Equation (10) is applicable for uniform heating as well as uniform temperature with the use of the appropriate values of A_F, C_F, A_N, and C_N. The choice of the mean temperature difference at $x = H/2$ is recommended since it yields coefficients A_F and A_N which differ negligibly from those for uniform temperature. The values of C_F and C_N do not differ greatly, and the correlation is quite insensitive to these values for the practical range of Pr. The compromise values of A_F, A_N, C_F, and C_N suggested at the bottom of Table 1 provide a good approximation for Nu for both boundary conditions and probably for intermediate ones.

Interpretation

Why did most prior investigators choose $n = 2$ or 4 instead of 3? Apparently their choice was based on some loose rationalization or cursory examination and was not tested critically. Equation (10) is actually quite insensitive to the choice of n in this range. A choice of $n = 4$ would result in a maximum decrease of 6% in the predicted value of Nu and a choice of $n = 2$ a maximum increase of 12%. The recommendation of McAdams (1942) to use the higher of the limiting coefficients is equivalent to letting $n \to \infty$ and results in a maximum underestimate of only 21%.

Equations (10), (11), and (12) indicate that the dimensionless group that determines the relative importance of free and forced convection in the laminar boundary-layer regime is $Ra/Re^2Pr^{4/3} = Gr/Re^2Pr^{1/3}$ in the limit of $Pr \to \infty$ and $Ra/Re^2Pr = Gr/Re^2$ in the limit of $Pr \to 0$. [These same groupings were noted by Acrivos (1966)]. Why have prior correlations generally been developed in terms of Gr/Re^2 instead of $Gr/Re^2Pr^{1/3}$, which is a reasonable approximation for most fluids? Apparently, this choice was based on incomplete rationalization or the confinement of attention to one fluid.

LAMINAR BOUNDARY-LAYER REGIME FOR OTHER SHAPES AND CONDITIONS

Correlations for horizontal cylinders, equivalent to Equations (7), (8), and (14) for vertical plates, have been developed by Churchill and Bernstein (1976), Churchill and Chu (1975*a*), and Churchill (1974). Insufficient theo-

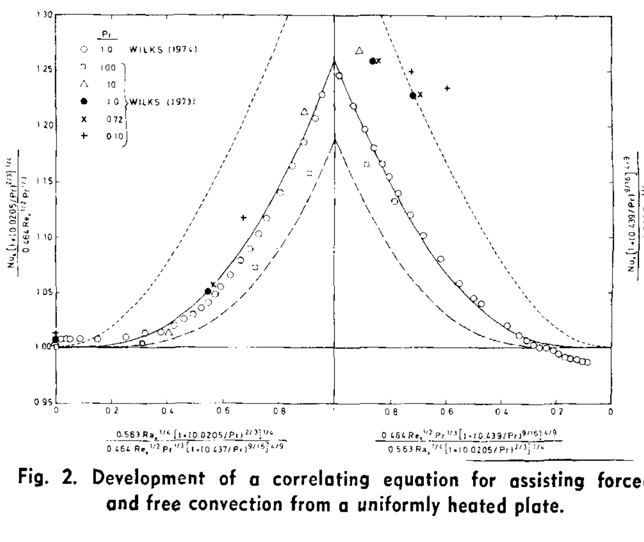

Fig. 2. Development of a correlating equation for assisting forced and free convection from a uniformly heated plate.

retical and experimental values are available to develop equivalent correlations for the effect of Pr on spheres. However, Churchill and Churchill (1975) have shown that correlations for free convection from vertical plates can be adapted as reasonable approximations for cylinders and spheres if πd and $\pi d/2$ are used as the characteristic dimensions in Nu and Ra. Krischer and Loos (1958) proposed a similar generalization for combined convection to many shapes but used $\pi d/2$ rather than πd for cylinders. However, they apparently made a numerical error in computing values of this characteristic dimension for cylinders.

In the absence of computed values, the same coefficients, C_F and C_N, will be postulated for spheres as for cylinders. Experimental values for A_F are listed in Table 1 for both cylinders and spheres. The use of the same characteristic dimension as above for free convection yields a value for A_F for spheres which is close to that of flat plates, but the value for cylinders is much higher. All of these values are summarized in Table 1. Together with Equations (10) (11), and (12) they constitute correlations for the laminar boundary-layer regime for all three shapes and for the two indicated boundary conditions.

The computed values of Chen and Mucoglu (1975) for combined convection to a vertical cylinder can be represented approximately by Equation (10) if Nu_x is replaced by $Nu_x - 0.88(x/d)$. The range of validity of this expression is uncertain, since their results were obtained by the method of local nonsimilarity, are limited to $x/d\ Re_x^{1/2} < 0.5$, $Gr_x/Re_x^2 < 2.0$, and $Pr = 0.7$, and were not tested with experimental data except for the limiting case of a flat plate. Equations (10), (11), and (12) are undoubtedly applicable for other geometrical configurations, such as inclined plates, wedges, and vertical cones, for other boundary conditions and for nonNewtonian fluids when limiting solutions for pure convection and computed values for combined convection become available. For example, Sparrow et al. (1959) give solutions for two particular wedge flows. Their values of Nu_x can be represented very closely by Equation (13) simply by choosing an appropriate characteristic length. However, these two conditions are insufficient to establish a general formulation.

The extension of the correlation to include forced convection oblique to or opposing free convection was attempted, but a competely successful generalization was not discovered. Weber (1976) asserts that such a generalization would be inappropriate on theoretical grounds, owing to the basic difference in the fluid motion.

CREEPING-FLOW REGIME (Re AND $Ra \rightarrow 0$)

Laminar boundary-layer theory eventually becomes invalid as Re and Ra approach zero. This limit occurs when the thickness of the boundary layer approaches the characteristic dimension of the surface in magnitude. The regime of lower Re and Ra is not of great practical importance for flat surfaces but may be for small horizontal cylinders (wires) and for small particles.

The correlating equation developed by Churchill and Chu (1975*a*) for free convection from horizontal cylinders indicates that laminar boundary-layer theory may be used with less than 10% error down to $Ra \cong 72\,000$ for $Pr \rightarrow \infty$ and to $Ra\,Pr \cong 40\,000$ for $Pr \rightarrow 0$. The correlating equation of Churchill and Bernstein (1977) for forced convection to cylinders indicates that the corresponding limits are $Re\,Pr^{2/3} \cong 50$ for $Pr \rightarrow \infty$ and $Re\,Pr \cong 36$ for $Pr \rightarrow 0$. These limits are presumed to apply approximately to combined convection and other geometries and hence to Equation (10).

A number of analytical solutions have been derived for pure forced convection at very small Re and for pure free convection at very small Ra. The latest and most reliable seem to be those of Nakai and Okazaki (1975), who obtained expressions which can be written as follows:

$$\overline{Nu_F} = 2\pi/\ln[16.32/RePr],\quad RePr < \pi/5 \qquad (15)$$

$$\overline{Nu_N} = 6\pi/\ln[4\,831(Pr + 9.4)^{1/2}/\overline{Nu}RaPr],\quad (RaPr)^3 < \pi/5 \qquad (16)$$

Equations (15) and (16) yield much larger values of $\overline{Nu}$ than the components of Equation (10) as Re and Ra approach zero. The experimental data of both Nakai and Okazaki (1975) and Gebhart and Pera (1970) for assisting convection indicate that a satisfactory correlating equation for the creeping-flow regime can be constructed by adding $\overline{Nu_F}$ and Nu_N to a high power such as 10. Such a large power implies that the higher of the values of $\overline{Nu}$ given by Equations (15) and (16) may also be used as a very close approximation. An alternative correlation can be constructed by first inverting and then raising Equations (15) and (16) to the eth power before combining. This procedure leads to a correlating power which can be rounded off to -2, hence to

$$\frac{4\pi}{\overline{Nu}} = \ln \frac{1}{\left(\dfrac{RePr}{16.32}\right)^2 + \left(\dfrac{NuRaPr}{4\,831(Pr+9.4)^{1/2}}\right)^{2/3}} \qquad (17)$$

The data of Gebhart and Pera indicate that L/d may be a significant parameter in the creeping-flow regime, particularly for large Pr. This dependence apparently stems from the three-dimensional motion generated at the ends of the wire.

Insufficient theoretical and experimental results have yet been obtained for other shapes to support the development of a generalized correlation for the creeping-flow regime.

Unfortunately, a considerable gap exists between the indicated upper range of validity of Equations (15) and (16) and the indicated lower limit of applicability of the components of Equation (10). One possibility would be to develop an expression for interpolation between these equations. However, the singularity in Equations (15) and (16) is a serious impediment. A simpler alternative is to approximate the limiting behavior in the creeping-flow regime by a constant value Nu_o and then to interpolate between Nu_o and the expressions for the laminar boundary-layer regime. The use of Nu_o has a sound theoretical basis and a value of π for pure conduction from spheres, with $\pi d/2$ as a characteristic length. The theoretical solution of King (1914) for forced convection to cylinders yields a value of 1.0 for Nu_o, with πd as a characteristic length, but is based on a questionable boundary condition. Churchill (1976) and Churchill and Chu (1975*b*) have shown that $Nu_o \cong 0.5$ provides a satisfactory empirical representation for the experimental data for both forced and free convection to flat plates. These authors and also Churchill and Chu (1975*a*) and Churchill and Bernstein (1977) have demonstrated that a simple sum of Nu_o and the laminar boundary-layer equations provides a satisfactory interpolating equation for pure forced and free convection from both plates and cylinders. Rather than add Nu_o to both components of Equation (10), it can be subtracted from Nu, yielding

$$\left(\frac{(Nu - Nu_o)f_F\{Pr\}}{A_F Re^{1/2} Pr^{1/3}}\right)^3 = 1 + \left(\frac{A_N Ra^{1/4} f_F\{Pr\}}{A_F Re^{1/2} Pr^{1/3} f_N\{Pr\}}\right)^3 \qquad (18)$$

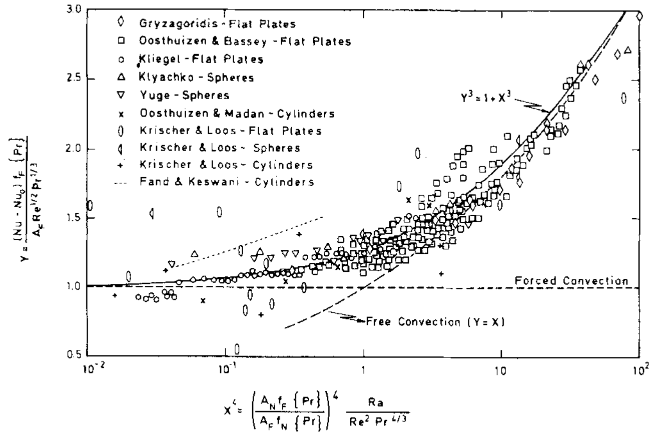

Fig. 3. General correlation for assisting forced and free convection.

The recommended values of Nu_o are included in Table 1.

Equation (18) is proposed as a practical representation for assisting convection for the entire laminar regime for all Pr, all geometries, and all boundary conditions, with the recognition that the values of Nu_o, A_F, A_N, C_F, and C_N and the expressions for l may be improved or others added as additional theoretical or precise experimental work is carried out.

For Re and Ra less than the values indicated as limits for Equations (15) and (16), the tenth root of the sum of the tenth powers of Equations (15) and (16) or Equation (17) may give more accurate values than Equation (18).

TURBULENT REGIME

The validity of the component parts of Equations (10) and (18) is presumed to be limited to approximately $Re < 10^4$ and $Ra < 10^9$, respectively, owing to the onset of turbulent motion. Accordingly, these equations might be presumed to have an approximate upper limit of validity corresponding to $(Re^{3/2} + Ra^{3/4}Pr^{-1}) = (10^4)^{3/2} = 10^6$. This limit seems worthy of experimental investigation.

COMPARISON WITH EXPERIMENTAL DATA

Representative experimental data for flat plates, spheres, and cylinders are plotted in Figure 3. The theoretical values plotted in Figures 1 and 2 are not included, since most of them would not show detectable scatter from Equation (18) with these more condensed scales. The experimental values for vertical plates scatter widely but rather randomly about Equation (18). The data for spheres and horizontal cylinders are more limited and scatter more widely, but again randomly. The results of Fand and Keswani (1973) for horizontal cylinders are plotted as a dotted line, since only smoothed, indirect values were given. The discrepancy may be due to this smoothing or to misinterpretation herein rather than to experimental error. The data of Garner and Hoffman (1960) for mass transfer from spheres were omitted since they converge to excessively high values of $\overline{Sh}$ as Re approaches zero. Likewise, the data of Pei (1965) for heat transfer from spheres were omitted, since they converge to excessively high values of $\overline{Nu}$ in both of the limits of pure convection. Regrettably, many of the published data, including the extensive values of Börner (1965), Narasimhan et al. (1967), Wilhelm (1969), and Brdlik et al. (1974) are given only in plots of combined variables which do not permit accurate conversion to the coordinates of Figure 3. A statistical analysis of the values plotted in Figure 3 does not appear to be justifiable in view of the uncertainty in the proper weighting factors, if any, to be applied to the several sets of data and to the different ranges of Re, Ra, and Pr.

In summary, Equation (18) is proposed as a working relationship for all shapes, boundary conditions, and Pr (or Sc) for the entire laminar regime. For the turbulent regime with $Re^{3/2} + Ra^{3/4}Pr^{-1} > 10^6$, it should serve as a lower bound for Nu. For very small cylinders, such that $(RaPr)^3$ and $Re\,Pr < \pi/5$, Equation (17) or the tenth root of the sum of the tenth powers of Equations (15) and (16) may give a more accurate value for Nu. In this regime, Equation (18) appears to be an upper bound and Equation (10) a lower bound.

ACKNOWLEDGMENT

The provision of tabulations of experimental values for flat plates by Professor P. H. Oosthuizen is greatly appreciated. The criticisms and suggestions of the anonymous reviewers were most helpful.

NOTATION

A_F = coefficient in correlation for forced convection, Equation (10)
A_N = coefficient in correlation for free (natural) convection, Equation (10)
B = function of Pr and L/d
C_F = central value of Pr (or Sc) for forced convection, Equation (6)
C_N = central value of Pr (or Sc) for free (natural) convection, Equation (7)
d = diameter, m
E = function of Pr, Equation (2)
F = function of Pr, Equation (2)
$f_F\{Pr\}$ = function of Pr for forced convection, Equation (11)
$f_N\{Pr\}$ = function of Pr for free (natural) convection, Equation (12)
g = acceleration due to gravity, m/s^2
Gr = $g\beta\Delta T l^3/\nu^2$ = Grashof number
H = height of plate, L
j = heat flux density, W/m$^2 \cdot$ s
k = thermal conductivity, W/m$^2 \cdot$ s $\cdot$ K
k' = component transfer coefficient, m/s
l = characteristic length given by Table 1 or subscript m
L = length of cylinder, m
n = exponent, Equation (1)
Nu = $jl/k\Delta T$ = Nusselt number
$\overline{Nu}$ = mean Nusselt number
Pr = ν/α = Prandtl number
p = exponent of Re, Equation (2)
q = exponent of Gr, Equation (2)
Q = graphical function of Pr determined by Sparrow and Gregg (1959)
Q_1, Q_2 = graphical functions of Pr determined by Eshghy (1964)
Ra = $g\beta\Delta T l^3/\nu\alpha$ = Rayleigh number
Ra' = $g\gamma\Delta\omega l^3/\nu\mathcal{D}$ = Rayleigh number for component transfer
Re = lu/ν = Reynolds number
Sc = $\nu/\mathcal{D}$ = Schmidt number
Sh = $k'l/\mathcal{D}$ = Sherwood number
u = velocity, m/s
x = distance up plate, m

Greek Letters

α = thermal diffusivity, m^2/s
β = volumetric coefficient of expansion with temperature, K^{-1}

γ = volumetric coefficient of expansion with composition
ΔT = temperature difference, K
$\Delta\omega$ = composition difference
ν = kinematic viscosity, m^2/s
$\varphi\{Pr\}$ = function of Pr, Equations (4) and (5)

Subscripts

d = based on diameter
F = forced convection
N = free (natural) convection
o = limiting value as Re and $Ra \rightarrow 0$
x = local value based on x

LITERATURE CITED

Acrivos, A., "On the Combined Effect of Forced and Free Convection Heat Transfer in Laminar Boundary Layer Flows," *Chem. Eng. Sci.*, **21**, 343 (1966).

Börner, H., "Über den Wärme- und Stoffübertragang um umspülten Einzelkorpern bei überlagerung von freier und erzwungener Strömung," *Ver. Deut. Ing. Forschungsheft*, No. 512 (1965).

Brdlik, P. M., I. A. Kozhinov, and A. A. Petrova, "Approximate Solution to the Problem of Local Heat Transfer at a Vertical Plate under Conditions of Laminar Mixed Convection," *Intern. Chem. Eng.*, **14**, 65 (1974).

Chen, T. S., and A. Mucoglu, "Buoyancy Effects on Forced Convection along a Vertical Cylinder," *J. Heat Transfer, Trans. ASME*, **97C**, 198 (1975).

Churchill, S. W., "Laminar Free Convection from a Horizontal Cylinder with a Uniform Heat Flux Density," *Letters in Heat Mass Transfer*, **1**, 109 (1974).

———, "A Comprehensive Correlating Equation for Forced Convection from Flat Plates," *AIChE J.*, **72**, 264 (1976).

———, and M. Bernstein, "A Correlation for Forced Convection from Gases and Liquids to a Circular Cylinder in Crossflow," *J. Heat Trans.*, **99C**, (1977), to be published.

Churchill, S. W., and H. H. -S. Chu, "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder," *Intern. J. Heat Mass Transfer*, **18**, 1049-1053 (1975*a*).

———, "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate," *ibid.*, in press (1975*b*).

Churchill, S. W., and R. U. Churchill, "A Comprehensive Correlating Equation for Heat and Component Transfer by Free Convection," *AIChE J.*, **21**, 604 (1975).

Churchill, S. W., and H. Ozoe, "Correlations for Laminar Forced Convection with Uniform Heating in Flow over a Plate and in Developing and Fully Developed Flow in a Tube," *J. Heat Transfer, Trans. ASME*, **95C**, 78 (1973*a*).

———, "Correlations for Laminar Forced Convection in Flow over an Isothermal Flat Plate and in Developing and Fully Developed Flow in an Isothermal Tube," *ibid.*, 416 (1973*b*).

———, "A Correlation for Laminar Free Convection from a Vertical Plate," *ibid.*, 540 (1973*c*).

———, and R. Usagi, "A General Expression for the Correlation of Rates of Transfer and Other Phenomena," *AIChE J.*, **18**, 1121 (1972).

Eshghy, S., "Forced-Flow Effects on Free-Convection Flow and Heat Transfer," *J. Heat Transfer, Trans. ASME*, **86C**, 290 (1964).

Fand, R. W., and K. K. Keswani, "Combined Natural and Forced Convection Heat Transfer from Horizontal Cylinders to Water," *Intern. J. Heat Mass Transfer*, **16**, 1175 (1973).

Garner, F. H., and J. M. Hoffman, "The Transition from Free Convection to Forced Convection in Mass Transfer from Solid Spheres," *AIChE J.*, **6**, 579 (1960).

Gebhart, B., and L. Pera, "Mixed Convection from Long Horizontal Cylinders," *J. Fluid Mech.*, **45**, 49 (1970).

Gryzagoridis, J., "Combined Free and Forced Convection from an Isothermal Vertical Plate," *Intern. J. Heat Mass Transfer*, **18**, 911 (1975).

Hatton, A. P., D. D. James, and H. W. Swire, "Combined Forced and Natural Convection with Low-Speed Air Flow over Horizontal Cylinders," *J. Fluid Mech.*, **42**, 17 (1970).

Jackson, T. W., and H. H. Yen, "Combining Forced and Free Convection Equations to Represent Combined Heat-Transfer Coefficients for a Horizontal Cylinder," *J. Heat Transfer, Trans. ASME*, 93C, 247 (1971).

King, L. V., "On Convection of Heat from Small Cylinders in a Stream of Fluid," *Phil. Trans. Roy. Soc. (London)*, **214A**, 373 (1914).

Kliegel, J. R., "Laminar Free and Forced Convection Heat Transfer from a Vertical Flat Plate," Ph.D. thesis, Univ. Calif., Berkeley (1959).

Klyachko, L. S., "Heat Transfer between a Gas and A Spherical Surface with Combined Action of Free and Forced Convection," *J. Heat Transfer, Trans. ASME*, **85C**, 355 (1963).

Krischer, O., and G. Loos, "Wärme- und Stoffaustausch bei erzwungener Strömung an Körpern verschiedener Form, Teil 1," *Chem. Ing. Tech.*, **30**, 31 (1958).

Lemlich, R., and R. Hoke, "A Common Basis for the Correlation of Forced and Natural Convection to Horizontal Cylinders," *AIChE J.*, **2**, 249 (1956).

Lloyd, J. R., and E. M. Sparrow, "Combined Forced and Free Convection Flow on Vertical Surfaces," *Intern. J. Heat Mass Transfer*, **13**, 434 (1970).

McAdams, W. H., ed., *Heat Transmission*, 2 ed., p. 217, McGraw Hill, New York (1942).

Merkin, J. H., "The Effect of Buoyancy Forces on the Boundary-Layer Flow over a Semi-Infinite Vertical Flat Plate in a Uniform Free Stream," *J. Fluid Mech.*, **35**, 439 (1969).

Morgan, V. T., "The Overall Convective Heat Transfer from Smooth Circular Cylinders," *Advances in Heat Transfer*, **11**, 199 (1975).

Nakai, S., and T. Okazaki, "Heat Transfer from a Horizontal Circular Wire at Small Reynolds and Grashof Numbers-II Mixed Convection," *Intern. J. Heat Mass Transfer*, **18**, 397 (1975).

Narasimhan, C., and W. H. Gauvin, "Heat and Mass Transfer to Spheres in High Temperature Surroundings," *Can. J. Chem. Eng.*, **45**, 181 (1967).

Oosthuizen, P. H., and M. Bassey, "An Experimental Study of Combined Forced- and Free-Convection Heat Transfer from Flat Plates to Air at Low Reynolds Numbers," *J. Heat Transfer, Trans. ASME*, **95C**, 120 (1973).

Oosthuizen, P. H., and S. Madan, "Combined Convective Heat Transfer from Horizontal Cylinders in Air," *ibid.*, **92C**, 194 (1970).

Pei, D. C. T., "Heat Transfer from Spheres under Combined Forced and Natural Convection," *Chem. Eng. Progr. Symposium Ser. No. 59*, **61**, 57 (1965).

Sparrow, E. M., and J. L. Gregg, "Buoyancy Effects in Forced-Convective Flow and Heat Transfer," *J. Appl. Mech.*, **26**, *Trans. ASME*, 81E, 133 (1959).

Sparrow, E. M., R. Eichhorn, and J. C. Gregg, "Combined Forced and Free Convection in Boundary Layer Flow," *Phys. Fluids*, **2**, 319 (1959).

Szewczyk, A. A., "Combined Forced and Free-Convection Laminar Flow," *J. Heat Transfer, Trans. ASME*, **86C**, 501 (1964).

Weber, M. R., "Letter to the Editor," *AIChE J.*, **22**, 207 (1976).

Wilhelm, R., "Der Stoffübergang fest-flüssig bei Einzelkörpern und Schüttungen in freier und erzwungener Strömung," *VDI Forschungshaft*, No. 531 (1969).

Wilks, G., "Combined Forced and Free Convection Flow on Vertical Surfaces," *Intern. J. Heat Mass Transfer*, **16**, 1958 (1973).

———, "The Flow of a Uniform Stream over a Semi-Infinite Vertical Flat Plate with Uniform Surface Heat Flux," *ibid.*, **17**, 743 (1974).

Yuge, T., "Experiments on Heat Transfer from Spheres Including Combined Natural and Forced Convection," *J. Heat Transfer, Trans. ASME*, 82C, 214 (1960).

Manuscript received June 23, 1976; revision received September 28 and accepted October 7, 1976.